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Y/3201

III Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper I

(Integration Theory and Functional Analysis-I)

Time : 3 Hours] [Max. Marks : 80

Note : *Attempt* ***Sections 'A', 'B', 'C'*** *according to the following instructions.*

Section 'A' **4 × 3 = 12**

(Very Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***three*** *lines.*

1. State Hahn decomposition theorem.

2. Define normed linear space.

3. Explain Bounded linear transformation.

P.T.O.

4. Define contraction mapping.

Section 'B'

(Short Answer Type Questions) **4 × 5 = 20**

Note : *Attempt all questions. Maximum word limit for each question* **150** *words.*

5. Let E be a measurable set of finite measure i.e., $0 < \mu(E) < \infty$ then E contains a positive set A with $\mu(A) > 0$.

Or

State and prove Lebesgue decomposition theorem.

6. A normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

Or

Let X be a linear space and let $\|\cdot\|_1$ and $\|\cdot\|_2$ be two norms defined on X then these norm an X are equivalent if and only if there exist positive constants m and M such that

$$m \| x \|_1 \leq \| x \|_2 \leq M \| x \|, \forall x \in X.$$

7. Let X and Y be normed linear space and T a linear transformation on X into Y then T is continuous either at every point of X or at no point of X. It is continuous on X if and only if there is a constant M such that :

$$\| Tx \| \leq M \| x \| \text{ for every } x \text{ in } X.$$

Or

In a finite dimensional normed linear space X prove that weak and strong converges coincide.

8. Let X be a metrizable space and $f : X \to R^*$ then f is lower semi-continuous if and only if epigraph f is closed in $X \times R$.

Or

Define convex function. Let X be a real vector space then $f : X \to R^*$ is convex if and only if epigraph f is convex.

Section 'C'

4 × 12 = 48

(Long Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* **500** *words.*

9. State and prove Fubini's theorem.

Or

State and prove Riesz representation theorem.

10. Let M be a closed linear subspace of a normed linear space X then show that the quotient space X/M is a normed linear space with norm :

$$\| x + M \| = \inf \{\| x + m \| = m \in M\}$$

further show that X is a Banach space then so is X/M.

Or

Let X be a normed linear space then show that the closed unit ball $B = \{x \in X : \| x \| \leq 1\}$ in X is compact if and only if X is of finite demensional.

11. Let $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$ then prove that l_p is isometrically isomorphic to l_q.

Or

Let X and Y be normed linear space then $B(X, Y)$ the set of all bounded linear transformation from X into Y is a normed linear space. Moreover if Y is a Banach space then $B(X, Y)$ is also a Banach space.

12. State and prove Picard's theorem.

Or

Derive the solution of Fredholm's Integral Equation.

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Y/3202

III Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper II

(Partial Differential Equation)

Time : 3 Hours]

[Max. Marks : 80

Note : *Attempt* **Sections 'A', 'B', 'C'** *according to the following instructions.*

Section 'A' $4 \times 3 = 12$

(Very Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***three*** *lines.*

1. Write statement of Euler-Poisson Darboux equation.

2. Write characteristics *ODE* of non-linear first order *PDE*.

P.T.O.

3. Write definition of Fourier transtorm and inverse Fourier transfrom.

4. Write definition singular perturbations.

Section 'B'

4 × 5 = 20

(Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question **150** words.*

5. Derive fundamental solution of Laplaces equation.

Or

Derive non-homogeneous problem of transport equation.

6. Derive characteristics for the Hamilton-Jacobi equation.

Or

State and prove the Euler-Lagrange equations.

7. Derive Barenblatt's solution to the porous medium equation.

Or

Derive Hopf-cole transformations.

8. Explain Cauchy data and non-characteristic surfaces.

Or

Write short note of real analytic functions.

Section 'C'

4 × 12 = 48

(Long Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question **500** words.*

9. Derive energy methods of Heat equation :

(a) Uniqueness 4

(b) Backwards uniqueness 8

Or

(a) Explain physical interpretations of wave equation. 4

(b) Derive solution for $n = 1$ by spherical means. 8

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10. Derive local existence theorem.

Or

State and prove the Lax-Oleinik formula.

11. State and prove the properties of Fourier transfrom.

Or

Explain : Hodograph transform and Legendre transform.

12. State and prove the Cauchy Kovalevskaya theorem.

Or

Derive Asymptotics for quadratic terms.

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Y/3203

III Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper III

[Programming in C (with ANSI Features)]

Time : 3 Hours] [Max. Marks : 50

Note : *Attempt* ***Sections 'A', 'B', 'C'*** *according to the following instructions.*

Section 'A' **4 × 2 = 8**

(Very Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***three*** *to* ***four*** *lines.*

1. Write in short about C programming.

2. How to declare integer data type.

3. Define 'for loop' statement in C.

P.T.O.

4. Write in short about increment operator.

Section 'B'

$4 \times 4 = 16$

(Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***150*** *words.*

5. Write any C programme with output.

Or

Compare Local and global variables.

6. Explain declaration of data types in C.

Or

Explain float data type in C.

7. Explain 'Do-While' loop in C.

Or

Write a brief note on break statements.

8. Explain Decrement operator with example.

Or

Explain relational operator with example.

Section 'C'

$4 \times 6\frac{1}{2} = 26$

(Long Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***500*** *words.*

9. Explain preprocessor in C.

Or

Explain function in C language.

10. Explain pointers in C.

Or

Explain different types of integer constants.

P.T.O.

4. Write in short about increment operator.

Section 'B'

(Short Answer Type Questions) $4 \times 4 = 16$

Note : *Attempt all questions. Maximum word limit for each question* **150** *words.*

5. Write any C programme with output.

Or

Compare Local and global variables.

6. Explain declaration of data types in C.

Or

Explain float data type in C.

7. Explain 'Do-While' loop in C.

Or

Write a brief note on break statements.

8. Explain Decrement operator with example.

Or

Explain relational operator with example.

Section 'C'

(Long Answer Type Questions) $4 \times 6\frac{1}{2} = 26$

Note : *Attempt all questions. Maximum word limit for each question* **500** *words.*

9. Explain preprocessor in C.

Or

Explain function in C language.

10. Explain pointers in C.

Or

Explain different types of integer constants.

11. Explain array in C.

Or

Explain Encryption and Decryption.

12. Explain memory operators in C.

Or

Write a program in C using logical operator.

B

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Y/3204

III Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper IV

(Operations Research-I)

Time : 3 Hours] [Max. Marks : 80

Note : *Attempt* ***Sections 'A', 'B', 'C'*** *according to the following instructions.*

Section 'A' **4 × 3 = 12**

(Very Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* ***three*** *lines.*

1. What is degeneracy in L.P.P. ?

2. Prove that dual of the dual of given primal is again dual.

3. Distinguish between Transportation and Assignment problem.

4. What is Network flow problem ? Illustrate with example.

Section 'B' **4 × 5 = 20**

(Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question **150** words.*

5. Discuss significance and scope of O.R. in decision making problem.

Or

Write a short note on sensitivity analysis.

6. Write a short note on parametric linear programming problem.

Or

Explain duality and dual simplex method.

7. Prove that the necessary and sufficient condition for the existance of feasible solution to a $m \times n$ transportation problem is :

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Or

Consider the problem of assignment five jobs to five persons. The assignment cost are given

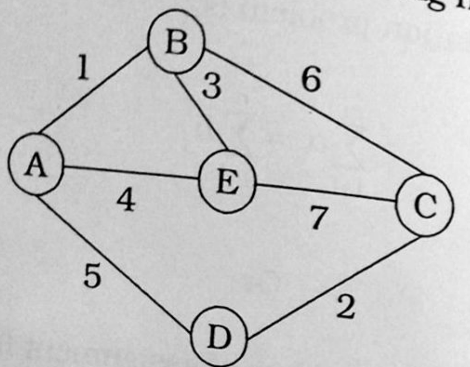
Persons \ Jobs	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine optimum assignment schedule.

8. Define slacktime, total float, free float, independent float, activity variance, project variance in cotext of network models.

Or

Use Dijkstra's algorithm to determine a shortest path from A to C for the following network



Section 'C'

4 × 12 = 48

(Long Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* **500** *words.*

9. Solve by Simplex method

$$\text{Max } Z = 5x_1 + 3x_2$$

Subject to constraints

$$x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Or

Use BIG-M method to solve L.P.P.

Maximize $\quad Z = 3x_1 + 2x_2$

Subject to constraints

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1,\ x_2 \geq 0$$

10. Use dual simplex method to solve L.P.P.

Minimize $\quad Z = x_1 + x_2$

Subject to constraints

$$2x_1 + x_2 \geq 4$$

$$x_1 + 7x_2 \geq 7$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Or

Consider the parametric linear programming problem

Maximize $\quad Z = 3x_1 + 2x_2 + 5x_3$

P.T.O.

Subject to constraints

$$x_1 + 2x_2 + x_3 \leq 430 + \mu$$

$$3x_1 + 2x_2 \leq 460 - 4\mu$$

$$x_1 + 4x_2 \leq 420 - 4\mu$$

and

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Determine the critical value of μ for which solution remains optical feasible.

11. Consider the following T.P.

Source	Destination				Availability
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	

Determine initial basic feasible solution using :

(a) North west corner method

(b) Vogels approximation method

Or

Solve the following Goal Programming Problem

Minimize

$$Z = p_1 d^- + p_2 d_4^+ + (2p_3 d_2^- + p_3 d_3^-) + p_1 d_1$$

Subject to constraints

$$x_1 + x_2 + d_1^- - d_1^+ = 10$$

$$x_1 + d_2^- = 6$$

$$x_2 + d_3^- = 8$$

$$d_1^+ + d_4^- - d_4^+ = 2$$

$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$$

12. Given the following information

Activity	Duration in days
0–1	2
1–2	8
1–3	10
2–4	6
2–5	3
3–4	3
3–6	7
4–7	5
5–7	2
6–7	8

(i) Draw the arrow diagram.

(ii) Identify the critical path and total project duration.

(iii) Determine total free and independent time.

Or

A project consists of eight activities with the following relevant information :

Activity	Immediate Predecessor	Estimated duration (days)		
		Optimistic	Mostlikely	Pessimistic
A	—	1	1	7
B	—	1	4	7
C	—	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

(i) Draw the PERT network and find out the expected project completion time.

(ii) What duration will have 95% confidence for project completion ?

(iii) If average duration for activity F increases to 14 days what will be its effect on the expected project completion time which will have 95% confidence ?

[For standard normal $Z = 1{\cdot}645$, area under curve from 0 to Z is $0{\cdot}45$]

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Y/3205

III Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper V

(Fuzzy Sets and Their Application-I)

Time : 3 Hours] [Max. Marks : 80

Note : *Attempt* **Sections 'A', 'B', 'C'** *according to the following instructions.*

Section 'A' **4 × 3 = 12**

(Very Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question* **three** *lines.*

1. Define alpha-level sets.

2. Define image of a fuzzy set.

3. Define composition of two fuzzy relations.

4. Define necessity measure.

P.T.O.

Section 'B' 4 × 5 = 20

(Short Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question **150** words.*

5. Normality and convexity may be lost when we operate on fuzzy sets by the standard operations of intersection and complement. Illustrate.

Or

Compute the scalar cardinality for the fuzzy set D on $X = \{0, 1, \ldots, 10\}$, where

$$D(x) = 1 - x/10 \text{ for } x \in \{0, 1, \ldots, 10\}$$

6. Illustrate extension principle for fuzzy sets.

Or

If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

7. Show that if R is a similarity relation, then each α-cut α_R is a crisp equivalence relation.

Or

Explain strong fuzzy homomorphism with suitable example.

8. Prove that plausibility measures are subadditive.

Or

Explain Dempster's rule of combination.

Section 'C' 4 × 12 = 48

(Long Answer Type Questions)

Note : *Attempt all questions. Maximum word limit for each question **500** words.*

9. Define the following with suitable examples :

(i) interval-valued fuzzy sets

(ii) fuzzy sets of types.

(iii) level 2 fuzzy sets.

P.T.O.

Or

State and prove the characterization theorem of *t*-norms.

10. Fuzzy numbers can be defined in a piecewise manner. Explain.

Or

Consider fuzzy sets *A* and *B* whose membership functions are defined by formulas

$$A(x) = x/(x+1) \text{ and } B(x) = 1 - x/10$$

for all $x \in \{0, 1, 2, \ldots, 10\} = X$. Calculate scalar and fuzzy cardinalities of *A* and *B*.

11. Explain the following with suitable examples of each :

(i) e-reflexive fuzzy relation

(ii) transitive fuzzy relation

(iii) antitransitive fuzzy relation

Or

If $S(Q, R) \neq \phi$ for the fuzzy relation equation

$$P_0^i Q = R,$$

then prove that $\hat{P} = (Q_0^{wi} R^{-1})^{-1}$ is the greatest number of $S(Q, R)$.

12. Show that the function *Bel* determined by $Bel = \sum_{B/B \subseteq A} m(B)$ for any given basic assignment *m* is a belief measure.

Or

Prove that a belief measure *Bel* on a finite power set $P(x)$ is a probability measure if and only if the associated basic probability assignment function *m* is given by $m(\{x\}) = Bel[\{x\}]$ and $m(A) = 0$ for all subsets of *X* that are not singletons.

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